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AN INTERPRETIVE STUDY OF PLATE AND SHELL BUCKLING

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AN INTERPRETATIVE STUDY OF PLATE AND SHELL BUCKLING

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The objective of the proposed research was to search the literature and locate all information relevant to plate and shell buckling; to collect the relevant journal articles, books, reports, theses, conference proceeding papers; to digest, summarize, organize and integrate the useful knowledge found and to make interpretative assessments; to determine problems where further research is needed; to produce a typed manuscript and ink drawings suitable for publishing a set of monographs; to clarify and unify the fields of plate and shell buckling.

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The work can be divided into three parts:

1. Identification and Procurement of References
2. Organization of the Literature
3. Monograph Writing and Preparation

A summary of each part is given below.

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I. IDENTIFICATION AND PROCUREMENT OF REFERENCES

Identification of references relevant to plate and shell buckling began with a thorough search through bound volumes of abstract journals. These included: Applied Mechanics Reviews, Scientific and Technical Aerospace Reports, Shock and Vibration Digest and Comprehensive Dissertation Abstracts. Approximately 3,000 references dealing with plate and shell buckling were identified in this way. Six automated machine searches (e.g., COMPENDEX, SCISEARCH, NTIS) were conducted, adding relatively few new references (<10%). Reference lists of related survey articles and books added additional sources.

Approximately 200 letters were sent to researchers world-wide, known to be recently active in plate and/or shell buckling, asking for relevant literature. Replies and useful information were received from about half. Finally, a large number of additional references were identified from the reference lists of publications already in hand. Altogether, a total of approximately 6,000 references dealing with plate and shell buckling were uncovered.

For each reference found, a 4 X 6" abstract card was prepared. A typical card contained on one side the typed bibliographical information in a form directly usable for subsequent manuscript typing (authors, title, journal, volume, page numbers, year) and, on the other side, one or more abstracts (xeroxed from the abstract journals), if available. The abstract cards provided a convenient, easily arranged catalogue of all known references, and were also very useful during the manuscript writing stage.

The majority of the relevant literature consists of papers published in the technical journals, and most of those were obtained by xeroxing bound volumes from the Ohio State University Libraries. Substantial additional xeroxing was done at the Battelle (Columbus), Air Force Institute of Technology (Dayton) and University of Colorado (Boulder) libraries. About 200 government reports were obtained through Aerospace Structures Information and Analysis Center (ASIAC) at Wright-Patterson Air Force Base.

II. ORGANIZATION OF THE LITERATURE

Because of the complexity of the subject and the enormous number of related technical references available, it was important to spend considerable time to determine a comprehensive, rational and understandable organization of the material. Plate buckling was organized into three parts as follows:

Part I. Classical Buckling Analysis

- 1.1 Governing equations
- 1.2 Rectangular plates
- 1.3 Stiffened rectangular plates
- 1.4 Circular and annular plates
- 1.5 Other shapes

Part II. Classical Complicating Effects

- 2.1 Elastic foundation and other displacement-dependent loads
- 2.2 Anisotropic material
- 2.3 Variable thickness
- 2.4 Shear deformation
- 2.5 Nonhomogeneous material

Part III. Nonclassical Phenomena

- 3.1 Postbuckling
- 3.2 Imperfections
- 3.3 Parametric excitation
- 3.4 Follower forces
- 3.5 Magnetoelastic buckling
- 3.6 Inelastic material

The above outline was developed and modified during the course of the project, and is believed to be a good one. Part I deals with plates governed by the classical buckling equation, which in rectangular coordinates is

$$D\nabla^4 w + P_x \frac{\partial^2 w}{\partial x^2} + 2P_{xy} \frac{\partial^2 w}{\partial x \partial y} + P_y \frac{\partial^2 w}{\partial y^2} = 0$$

where w is the transverse displacement, D is the flexural rigidity, P_x and P_y are compressive inplane forces (per unit length) and P_{xy} is inplane shearing force, along with appropriate linear equations representing the boundary conditions. For Part II the governing equation (1) is generalized in a different manner for each topic, but the overall problem remains a linear eigenvalue problem. Inclusion of the topics listed under Part III in a buckling study could be debated, for the traditional understanding of buckling as a bifurcation phenomenon associated with a linear eigenvalue problem is violated by them; yet they are important considerations in the theoretical and practical understanding of the significance of buckling.

Each section of each part listed above was subdivided further. For example Section 2.1 (Rectangular Plates) was organized as follows:

2.1 All Edges Simply Supported

- 2.1.1. Uniform normal stresses (σ_x and σ_y constant, $\tau_{xy} = 0$)
- 2.1.2. Uniform shear stress (τ_{xy} constant, $\sigma_x = \sigma_y = 0$)
- 2.1.3. Combined normal and shear stresses (σ_x , σ_y , τ_{xy} all constant)
- 2.1.4. Stress varying linearly in the direction of loading
- 2.1.5. Linearly varying bending stress
- 2.1.6. Combined bending and shear stresses
- 2.1.7. Combined bending and transverse stresses
- 2.1.8. Other stress variations
- 2.1.9. Point loads
- 2.1.10. Partial edge loads
- 2.1.11. Other loadings

2.2 Other Cases with Two Opposite Edges Simply Supported

- 2.2.1. SCSC
- 2.2.2. SCSS
- 2.2.3. SCSF
- 2.2.4. SSSF
- 2.2.5. SFSF

2.3 All Edges Clamped (with subsections similar to those of 2.1)

2.4 Other Classical Edge Conditions

- 2.4.1. CCCS
- 2.4.2. CCCF
- 2.4.3. CCSS
- 2.4.4. CCSF
- 2.4.5. CCFF
- 2.4.6. CSCF
- 2.4.7. CSSF
- 2.4.8. CSFF
- 2.4.9. CFCF
- 2.4.10. CFSF
- 2.4.11. CFFF
- 2.4.12. SSFF
- 2.4.13. SFFF
- 2.4.14. FFFF

2.5 Nonclassical Support Conditions

- 2.5.1. Elastic edge supports
- 2.5.2. Discontinuous edge conditions
- 2.5.3. Point supports

2.6 Internal Holes and Cracks

- 2.6.1. Circular holes
- 2.6.2. Other holes
- 2.6.3. Cracks or slits

What the above outline for Section 2 shows is the further detail which must be considered in the complete organization of available material. Thus, it is seen, the next level of organization is by edge constraint conditions, followed by type of loading. In the case of a plate having all edges simply supported (SSSS) considerable information was found for each of the eleven categories of loadings listed. For all sides clamped less was found, but sufficient to warrant its separate consideration. The cases of subsection 2.2 all have exact solutions for constant σ_x and/or σ_y , and thus were also set apart.

Shell buckling required one higher level of organization beyond that of plates. That is, shells were first divided by curvature, namely:

- 1. Circular cylindrical
- 2. Noncircular cylindrical
- 3. Conical
- 4. Spherical
- 5. Other shells of revolution
- 6. Other curvatures

In addition, for each type of curvature, shells can usually be classified as closed (e.g., a complete sphere) or open (e.g., a spherical cap). For each of these, the further considerations of edge constraint conditions, loadings, etc., as outlined above for plates, can be made.

Thus, many hundreds of hours were spent in looking through abstracts of references (or the references themselves) to determine where in the subject outline each reference fit. In many cases a single reference had useful information relevant to two or more sections of the outline.

III. MONOGRAPH WRITING AND PREPARATION

After the number of references relevant to plate and shell buckling were determined (~6,000), it was decided that at least two monographs, if not more, would be necessary to cope with the enormous volume of information available. Indeed, it was questionable whether the relatively smaller subject of plate buckling (~1,500 references) could be thoroughly treated in a single monograph of reasonable size. On the other hand it was felt that no summarization should be done which was not reasonably thorough. The designer, engineer, researcher or teacher who uses this type of monograph should be able to feel confident that, as of a certain date, the vast majority of what is known about the subject will be summarized in this single place.

Before the actual writing could begin, it was necessary to determine the procedure by which the monographs would be published. Three methods of publication were considered feasible:

1. U.S. Government Printing Office
2. Commercial book publisher
3. Private printing and distribution

Each method had its disadvantages. The major disadvantage of the first is the apparent capriciousness of GPO in determining the number of copies to print, and whether to reprint. The monographs on plate and shell vibrations (References 1 and 2) were printed in 5,000 copies each. The first was sold out by the Superintendent of Documents in one year, the second in two years, but GPO refused to reprint, citing "lack of demand." Both monographs were typeset, which required going through laborious editing of page proofs and galley proofs. It was therefore hoped that the present publication could be made from camera-ready sheets. Printing by GPO results in very little advertisement; almost everyone discovered the previous monographs "by accident." A commercial book publisher would provide good publicity, but the cost per copy could be high. Private printing made from camera-ready sheets could result in the least expensive copies. But a considerable sum of money would have to be invested in the printing, and no suitable way of doing this was found. On the basis of all considerations, it was decided early in the project that a commercial publisher would be most desirable. It was hoped that the use of camera-ready sheets would keep copy cost sufficiently low to permit individuals to have it for their bookshelves. Thereby the monographs would provide their greatest value.

Ten well-known technical publishers were selected. A letter was sent to each asking of their possible interest. Accompanying the letter was a rather large package of supplementary material directly applicable to the first monograph (Buckling of Plates). This package included: background information (other available summarizations, marketability, usage, scope, etc.), an outline, and a xerox copy of Chapter 2 of Vibration of Plates (Reference 1). After considerable time (approximately one year) two publishers were found to be sufficiently interested to offer a contract. One was rejected (Pergamon Press) because they wanted to set the book in type, yielding a prohibitively

high price, and because of a clause requiring the author (or the research contract) to purchase 200 copies for his own distribution. The other (Academic Press) was found suitable and a contract for Buckling of Plates was signed. The agreement now calls for the photoreproducible copy to be ready in June, 1984.

Writing of the monograph was done subsection-by-subsection. Thus, all the references (typically 10 to 20) dealing with a particular subsection were extracted from the main collection and were examined individually to determine, for example, the type of results obtained (theoretical versus experimental), the method of theoretical solution followed, the accuracy of numerical results presented, and the scope of parameters for which results were given (e.g., plate aspect ratios, loading ratios, Poisson's ratio). Useful results were integrated into each subsection by written text (e.g., description of methods used), tables (nondimensional buckling parameters for the various geometric parameters) and figures (descriptive, buckling parameter curves, mode shapes).

Typing of the manuscript text was done on a Xerox 860 word processor. This word processor was acquired by the Department of Engineering Mechanics approximately two years after the start of the present contract, and considerable time was required by office personnel to understand its operation. The advantages of using a word processor for manuscript preparation are usually outstanding. Changes in the text can easily be made. Furthermore, margins may be automatically "justified" (i.e., terminal letters of each line ending at the right-hand margin, instead of staggered, print wheels with proportional, instead of fixed, letter spacing) may be used, and printing is typically very clear. These latter advantages are particularly desirable in the preparation of a book manuscript by means of camera-ready sheets. However, after considerable effort was made, Xerox 860 was found to be poor when Greek symbols or equations were required. This required using a two-pass printer, with associated misalignment problems, and using two print wheels of different pitch.

Most of the figures involved curves taken directly from the various reference sources. These were typically drawn by photographically enlarging the original source, tracing the curves onto 8 1/2 X 11" vellum sheets, and adding abscissae, ordinates, or other labeling which is consistent with that used elsewhere in the monograph. Numerical data for all tables were converted to a common set of parameters used throughout the work.

IV. ADDITIONAL WORK PERFORMED, NOT PLANNED FOR IN THE ORIGINAL PROPOSAL

The number of relevant references found (~6,000) were approximately four times the number anticipated (~1,500). This required a corresponding increase in the number of hours spent in all phases of the work.

Stiffened plates and shells were omitted from the previously written monographs on vibrations and shells, and this was the original intent here. Soon after beginning work, the Principal Investigator was asked by ONR technical staff to expand the scope of the present project to include them. This added many hundreds of additional references to the total.

In spite of the vast literature found, the very important subject of classical rectangular plate buckling was found to be seriously lacking or even nonexistent for many of the combinations of edge conditions and loadings. To remedy this, an analytical procedure based upon the Ritz method was developed, and a corresponding computer program written, to obtain accurate results for all possible combinations of edge conditions (e.g., CSFF) and all possible loadings (σ_x , σ_y , τ_{xy}) which are constant or linearly varying throughout the plate. Buckling parameter results for a reasonable range of plate aspect ratios ($a/b = 0.5, 1, 2$) were obtained, and are incorporated into the monograph, thereby filling many serious information gaps which would otherwise exist. The method and representative results were developed as a M.S. thesis investigation by Mr. Chandru Kalro, and will also be published in a suitable journal.

Three published papers resulted from the contract (References 1-3). All three were survey papers summarizing recent research in the vibrations and buckling of plates. The first two, containing 315 references altogether, were combined into a technical report for the project (December, 1981). The last paper, which contained 116 references, was also put into technical report form (December, 1982). Approximately 200 copies of each report were sent to those listed on the ONR distribution list.

V. REFERENCES

1. Leissa, A.W., Vibration of Plates, NASA SP-160, U.S. Govt. Printing Office, 353 pp. (1969).
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3. Leissa, A.W., "Plate Vibration Research, 1976-1980: Classical Theory," The Shock and Vibration Digest, vol. 13, No. 9, pp. 11-22 (Sept., 1981).
4. Leissa, A.W., "Plate Vibration Research, 1976-1980: Complicating Effects," The Shock and Vibration Digest, vol. 13, No. 10, pp. 19-36 (Oct., 1981).
5. Leissa, A.W., "Advances and Trends in Plate Buckling Research," presented at the Symposium for Advances and Trends in Solid and Structural Mechanics, Washington, DC, October, 1982, published in "Research in Structural and Solid Mechanics - 1982," NASA CP-2245, edited by J.M. Housner and A.K. Noor, 441 pp. (1982).